**Question 1**

A. D.

B. E.

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**Question 2**

We prove by complete induction that

T(n) <= cn, for all n >= 1

n = 1: we must prove

a <= c

which can be done by setting c = a.

n = 2: we must prove

a <= 2c

which can be done by setting c = a/2.

Now suppose that n >= 3 and that for 1 <= m < n, we have T(m) <= cm. Note that n/2 and n/3 are both between 1 and n-1, so the IH applies on T(n/2) and T(n/3). (As usual in this course, we are ignoring floors.)

T(n) = T(n/2) + T(n/3) + 1

<= cn/2 + cn/3 + 1, by using IH twice

= cn(5/6)+1

= cn-(1/6)cn+1

<= cn, as long as

-(1/6)cn+1 <= 0

or

-(1/6)cn <= -1

or

-cn <= -6

or

cn >= 6

or

c >= 6/n

As 6/n is decreasing, we can satisfy this last inequality by letting c = 2.

The proof is complete, with n0=1 and c = max(a, 2).

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**Question 3**

A.

Pre: a >= 0, b >= 0,

Post: return a^b

B. "b" and "a+b"

b decreases, and a stays the same. So b is fine by itself, and decreases by the same amount if we use a+b instead.

C.

Path 1: b == 0. Then, 1 is the correct return value, because x^0 == 1 for any x.

In path 2 and 3, we have this recursive call:

x = power(a, b//2)

As b > 0 here, we have b//2 < b, so b can be used as the size of the recursive call. The first parameter doesn't change, and if b > 0 then b // 2 >= 0, so the preconditions hold on the recursive call. Therefore we conclude that x = a^{b//2}

Path 2: b is odd. In this case, we have

x = a^{b//2} = a^{b/2-0.5}

and so

x\*x\*a

= a^{b/2-0.5} \* a^{b/2-0.5}\*a

= a^{b-1}\*a

= a^b

Path 3: b is even. In this case, we have

x = a^{b//2} = a^{b/2}

and so

x\*x

= a^{b/2} \* a^{b/2}

= a^b

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**Question 4**

A. x >= 0

B.

e.g.

2x+j

2x+j-1

2x+j-2

But not 2x+j-3